

VALUE CHAIN MODELLING TO EVALUATE GEOMETALLURGICAL RECOVERY FACTORS

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ABSTRACT

Many recently developed tests permit faster characterisation of a broad range of rock properties in much greater detail than was possible even a few years ago. These data are being used to build geometallurgical models, and predict processing response properties on a spatial basis.

Scenario Based Project Evaluation (SBPE) is a stochastic approach to modelling the mining project value chain, which incorporates sources of uncertainty, multiple operational configurations, and future scenarios to generate a plausible range of project outcomes. This approach facilitates the use of the richer information provided by geometallurgical models and allows for ranking of sources of up and downside risk to the project. The approach also provides a platform for exploring the impact of numerous risk mitigation strategies, supporting robust decisions, increasing project resilience and improving the probability of mining project success.

Developing and using value chain models to evaluate geometallurgical recovery factors will markedly improve the probability of developing resilient project designs, making robust project decisions and hence greater project success by:

- Facilitating rapid processing of large and potentially un-wieldy geometallurgical models, making the SBPE methodology far more accessible;
- Providing a means to assess the impact of mineralisation variability on process performance; and
- Ensuring that the impacts of uncertainty, variability and constraints in project evaluation are explicitly quantified and thus suitable addressed.

INTRODUCTION

The overall imperative of any business is to continue as a profitable concern. In mining, this requires the acquisition of potentially profitable resources, designing and executing a profitable configuration and operation of existing mining projects, and successful disposal of projects that have declining profitability. The management of mining projects requires that decisions be based on ‘true knowledge’. True knowledge according to Demming (1986) begins with understanding the system, requires a quantitative understanding of the impact of variance, addresses the limits to knowledge and is tempered by insights into human nature and behaviour.

Porter (1985) suggests a value chain model can represent the value accrual in the system. A key concept in the value chain model is that, although the financial transaction (point of cash flow) in a value chain is easy to identify, it is not the only value-adding step. Value is created throughout the value chain even though in some steps there is no readily identifiable cash transaction. The value chain approach presents an opportunity to explore the first three requirements for true knowledge as posited by Demming (Demming, 1986) i.e., understanding the system, understanding the impact of variance on the system and exploring the impact of uncertainty. Scenario Based Project Evaluation (SBPE) is an approach to mining value chain modelling depicted in Figure 1, which can be used to explore, quantitatively, the interactions between the uncertain and variable properties of the mineralisation, the constraints and the flexibility engineered into the mining project (Vann et al., 2012). It also presents mining firms with a platform to explore the evolution of important, project relevant, aspects of the future (prices, taxes, demand, revenues etc.) by using the SBPE approach within a scenario framework (van der Heijden, 2005).

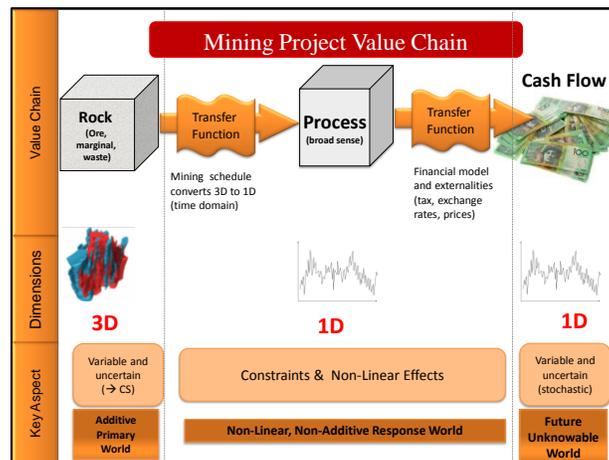


Figure 1: A value chain model for a mining project adapted from Vann et al (2012)

The value chain model used in SBPE is both holistic – covers the full extent of the value chain from source to final product – and integrated in that it correctly models the interaction of variability, constraints and feedback loops. This requirement applies to flows of goods, cash and information through the value chain at the scale relevant to the decisions being considered. Several iterations of project configuration, termed *alternatives*, and several permutations of future outcomes, termed *scenarios* can be evaluated by simulating the total value chain model.

The outputs from this process, which by design is *not* optimisation, can be used to search for plateaus of value where the configuration might not yield the highest financial return (e.g., NPV, IRR, payback) but on average the most unvarying, or resilient, financial return in the face of pertinent sources of uncertainty and variability. These sources of uncertainty and variability typically arise from the mineral resource, process operation and future operating contexts.

The SBPE outputs should assist in selecting the configuration that is most robust in terms of the expected project returns and will also help to improve the understanding of the thinking that underpins the sources of value in the project. This systems thinking process is akin to synthesis rather than analysis (Meadows, 2008, Senge, 1994) in that outcomes result not just from the summing of the function of each of the systems components but rather of the dynamics of the entire system. It gives experienced professionals an opportunity to express, in a quantitative systematic framework, the range of expected project outcomes for a given configuration or alternative. This approach has the added benefit of being able correctly to capture and reflect the incremental value that can be attributed to activities that reduce uncertainty of project outcomes (e.g., additional sampling, drilling, geometallurgy test work) and increase the system resilience to changed inputs and or operating strategies.

“Essentially all models are wrong but some are useful” (Box and Draper, 1987). Models used in the value chain are by necessity wrong in that they are not the *real* value chain, but aim to reflect the patterns that explain how value is either increased or decreased through the value chain. These patterns include counterintuitive outcomes that result from the non-linear and biasing impacts of the interaction of variability with constraints in the system. Ideally the model should provide a platform for quantitative feedback, over the life of the operation, of the interaction of the variable and uncertain mineralisation with the flexibility and constraints that are embodied in the project configuration. This quantified view of the operation can be used to identify the potential impact that flexing the project’s design and execution might have, and to ‘re-perceive’ the value (or not) of the envisaged project.

A key aspect of using a model to aid in understanding a project is to incorporate the main sources of uncertainty and allow these to interact at the correct scale (both spatial and temporal) with the flexibility, stocks and constraints that are a function of the design and operation of the project. Three sources of project uncertainty can be distinguished and which need to be incorporated in different ways into the value chain model:

- 1- **Spatial uncertainty** – i.e. the resource model. Additional sampling can reduce this uncertainty. It can be quantified by a suite of spatial simulations of the mineral resource. These need not be restricted to grades and can include, inter alia, mineralogy, geometallurgical parameters and deleterious components.
- 2- **Operational uncertainty** – this uncertainty arises from the interaction of the mineralisation and the operation of the mining and processing plant. Flexibility and constraints are set by the process configuration and these can be adapted by design and operational strategy. This uncertainty can be replicated by stochastic process simulation.
- 3- **Future uncertainty** – the context in which the project will run, this cannot be predicted with any reliability, one functional method is to use the Scenarios approach developed by Wack, Sunter and van der Heijden (Van der Heijden, 2005). In this approach it is possible to set up several plausible internally consistent futures in which the project could operate and then use these to calibrate reasonable values for the parameters of relevant distributions.

METHODOLOGY

The approach suggested here is based on the development of an integrated value chain model that includes a model of the mineral resource, models of the mining and processing activities (i.e., the value chain) and models of the future context in which the operation might operate (scenarios). These models are linked together to provide a platform for simulating the operation and evaluating how mineralisation uncertainty and variability propagate through the value chain over the life of the operation. The objective of this process is to determine more robust development and operational strategies. A quantitative analysis of the interaction of mineralisation characteristics with mining and processing provides insights that can be used to improve the probability of project success in a number of ways. This section describes the three main models that are considered, and the way in which they are linked and simulated.

Models of Mineralisation Uncertainty

Data derived from samples taken from a mineralisation (and its environs) can be used to generate estimates of values at unsampled locations. Linear estimation techniques, such as kriging (Journel and Huibrechts, 1978) generate models that are smoother than reality but are useful and appropriate for developing ultimate pit designs and long-term mining sequences. However, smoothed estimates are likely to underestimate variability especially in the shorter term. Kriged models (including non-linear kriging) also enable quantification of mineralisation uncertainty.

An alternative approach to estimating spatial models of mineralisation characteristics is to use geostatistical simulation, which generates unsmoothed spatial models. There are several algorithms that can be used to generate geostatistical simulations of spatial mineralisation characteristics, including sequential Gaussian simulation ((Deutsch, 1992), the turning bands method (Journel, 1994) and pluri-gaussian methods (Dowd et al, 2003). Within a given domain and at a specified scale, geostatistical simulation reproduces the spatial variability (via the variogram) of the data and the distribution (histogram) of the data. Conditional geostatistical simulation also honours actual data values at sampled locations. It is possible to generate multiple realisations of the mineralisation, each with the variability that will be encountered when mining. A sufficiently large set of realisations constitutes a model of spatial mineralisation uncertainty, both globally and locally. It is also possible to simulate the geometry of the envelopes into which the mineralisation and other pertinent properties are simulated.

Primary rock properties (such as porosity and density) are generally additive and, consequently, can be easily estimated and spatially simulated. This is not true for metallurgical response properties (such as permeability, crushed rock density distribution), which are often inherently non-linear and non-additive. Coward et al (2009) give a framework for classification of geometallurgical parameters. Geometallurgical modelling should proceed by building spatial models of primary rock properties which ‘drive’ key processing responses, and thus allow correct handling of non-linearity and non-additivity. Spatially informed prediction of processing responses necessitates modelling the linkage of primary properties of the *in situ* mineralisation (e.g., mineralogy and grades) with metallurgical performance measures at various scales (e.g., recovery and throughput). The concept of scale in this context must encapsulate the physical scale of primary sample extracted (the sample support), the scale of the selective mining unit (SMU) used in geostatistical simulations (the SMU support) and the scale of the metallurgical tests from which the responses are inferred (what metallurgists call ‘scale’).

Figure 2 shows a plan view of a mineralised body with thickness plotted. The left image shows

a geometry that is far more continuous and less variable than the realisation on the right.

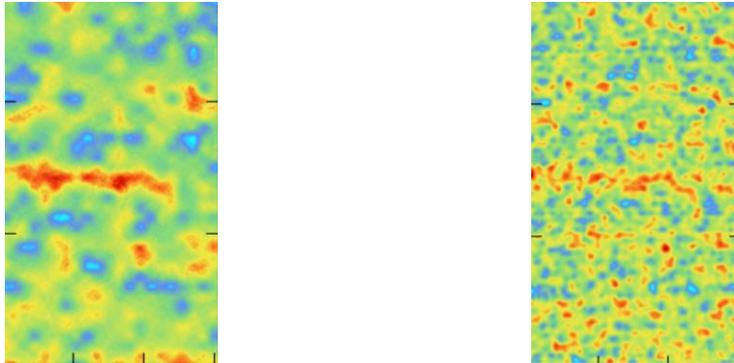


Figure 2: Images showing the outcomes of spatial simulation of mineralisation thickness, thicker areas shown in lighter colours and thinner areas in darker colours

Models of Mining and Processing

Development of an ultimate pit shell, the mining sequence and the mining schedule require the consideration of spatial mineralisation properties, process configuration and economic parameters. The ‘optimal’ mine plan yields the highest net present value (or the maximum of some other financial indicator). This requires the aggregation of the mined material into larger parcels to make the optimisation process tractable. Thus a smoothing set is required (if the inputs are kriged blocks this is an additional smoothing; Vann et al, 2011). Conventional mining optimisation assumes that the input parameters are all deterministic, i.e., exactly known.

Furthermore, the solution of the long-term mine plan usually defines a number of ore parcels (commonly shapes representing the geometry of annual excavations termed annual ‘push backs’ or ‘shells’) that are to be mined in a sequence over an extended period. Once the mine is operational this long-term optimal mine plan usually forms the basis for shorter term mine planning within each sub-parcel to achieve a tonnes and grade profile in a shorter time horizon (e.g., months). This short-term mine plan is necessary for operational planning and control.

Scenario-based project evaluation (SBPE) requires the ability to translate a mine plan at any resolution, usually annual or at most quarterly, into shorter time frames and to sequence blocks in a logical manner. Although there are approaches to optimisation of the mine plan under conditions of uncertainty (Dimitrakopoulos et al., 2002, Dowd and Dare-Bryan, 2004), currently sub-scale optimisation is not practically applicable for real time optimisation of the sequence of processing of block models. This is due to the fact that there can be in excess of several million blocks (in large deposits hundreds of millions) each containing multiple (50-300) realisations for several grade and physical attributes.

In SBPE once the optimal long-term mine plan has been derived, producing monthly, quarterly or annual parcels of blocks to be mined, these are sequenced using geometric constraints. In some cases it is possible to apply a selection algorithm to find the ‘best’ blocks in a way that approximates an optimal blend or sequences. An example of this is the grade stress algorithm developed by Everett (Everett, 2010) for iron ore mines.

Stockpiles not only provide buffers to manage feed rate variability to processes, but, if combined with correct operational strategy, can also provide an opportunity either to blend or

segregate materials. Using conditional simulation as inputs provides high-resolution models that reflect realistic variability at the SMU scale. It is thus possible to model realistically the impact of stocks and stockpiling strategies on the properties of material moving between processes. These impacts are often understated, and can lead to biased results, especially when applying transfer functions to the averages of large parcels of ore. This effect is demonstrated in Figure 3, where product grade targets are based on processing mined grade through a regression function. The dark solid line shows the target derived by processing the annual average feed grade through a regression function; the dark dashed line shows the targets calculated using the regression model on monthly-mined grade. Not only is there a difference between the two methods but the direction of the bias changes depending on the input grades.

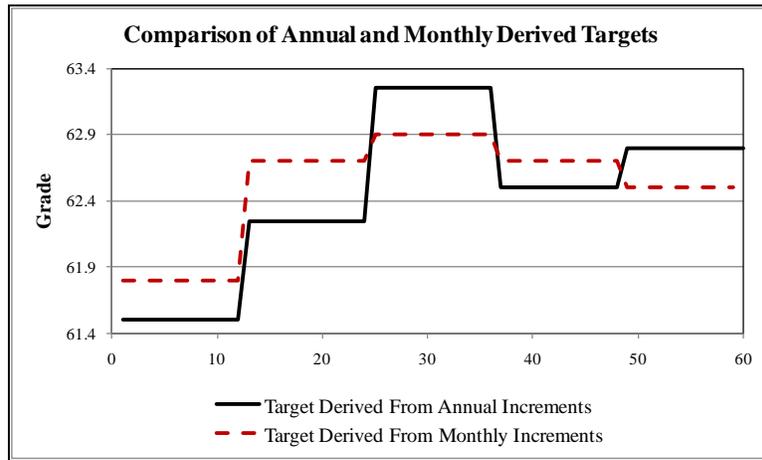


Figure 3: A chart showing the difference in target derived from processing annual averaged mined grades through a regression function (Black Line) and deriving the annual targets based on a regression of monthly-mined grades.

Process simulation of an operating plant requires the use of calibrated unit process models. A popular approach includes the use of population balance models where the mass of material in the circuit is maintained. (King, 2001, Napier-Munn, 1999) The model iterates until the differences in mass flows entering and leaving units in the flowsheet converge to a specified minimum. This process is computationally intensive and it is currently unrealistic to run these models to convergence for every block when block models contain in the order of, say, 250 000 blocks. The approach suggested here is to evaluate the process response to a number different combinations of rock property inputs (e.g., rock strength parameters, different feed blends) until sufficient data are acquired to model the relationship between input primary variables and the process responses of interest (Coward et al., 2009). Pragmatically, these can often be reasonably modelled by linear ($y = mx + c$) or second-order quadratic functions. Fitting these parameters is more generically known as parameterisation of inverse problems. (Tarantola, 2005).

The left hand image in Figure 4 shows results of simulating the diamond recovery process response using a process flow-sheet simulator. The least squares regression line for the data is also shown. The SBPE process model assumes a process transfer function of the form $y = mx + c$, where y is the output value of interest, m is a slope parameter and c is a constant used in the model. For each of the input parameters used it is possible to estimate the standard error. The standard error is a function of sample size and the variability observed in the data or

simulations. The standard error can be used to provide information on how the re-estimated parameter may change with a new sample from the same underlying distribution. In summary, the data from the plant were used to derive a number of points to generate a relationship between % kimberlite and % recovery. By removing one point and re-estimating m and c ('bootstrapping') it was possible to derive a set of m and c parameters. Using these multiple values of m and c it was then possible to calculate a standard error of the mean value for m and for c by dividing the standard deviation of the values thus obtained by the sample size used in the bootstrapping procedure. Using this standard error, it was possible to draw, at random, values for m from a normal distribution with a mean equal to the m estimated with all the data and a standard deviation equal to the standard error. A similar procedure was followed for the c value. Although we simulated the m and c parameters independently there is a need in future work to account for any correlations that might exist between these parameters.

The resulting relationships that can be used when running the value chain model are depicted in the right hand chart in Figure 4.

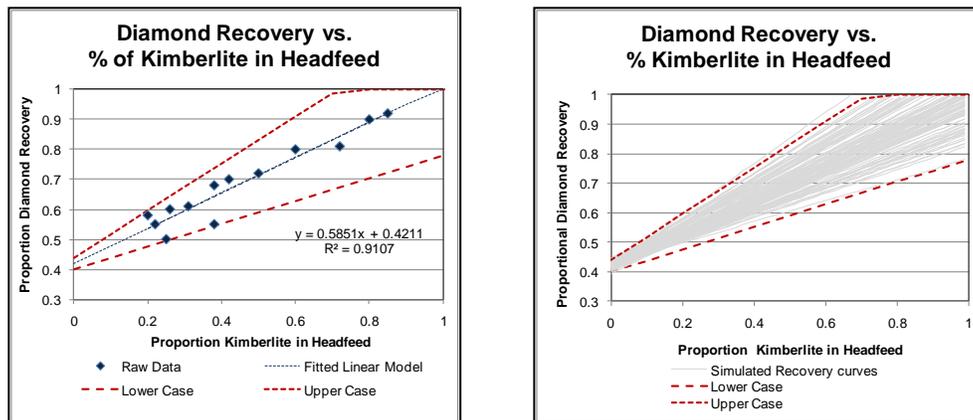


Figure 4: Plot of derived relationship between proportion kimberlite and fractional diamond recovery (LHS) and simulated recovery curves (RHS)

In this way, it is possible to generate many such curves for each of the required processes that can be sampled randomly for each iteration of the model. In this way, operational uncertainty is introduced into the recovery model, in a way that is related to the variability and number of observations that have been obtained. Using these models, it is possible to simulate the process operation and recovery efficiency on a block-by-block basis. Figure 5 depicts the daily production statistics, including the tonnes mined, carats recovered, end of day stockpile level, the blocks mined and the diamond recovery factor.

Models of Future Context

There is a rich literature and tradition of industrial practice in which problems involving high degrees of uncertainty have been dealt with from a scenario viewpoint. This approach, variously called 'scenario planning', 'scenario analysis' and 'scenario thinking' endeavours to preserve and explore the consequences of uncertainty rather than ignore it (Ilbury and Sunter, 2001, van der Heijden, 2005, Ramirez et al., 2010).

Mining companies have varying degrees of choice and control over configuration of the mining, blending, and mineral processing infrastructure and associated policies (Vann et al., 2012).

However, firms generally have *no* control over externalities, which are very significant drivers of economic success and sustainability performance. For many of these externalities (exchange rates, interest rates, market prices) stochastic models, based as they are on distributions and covariances that are necessarily obtained from past behaviours, (Biger and Hull, 1983) may be at least incomplete and at worst misleading and biased. The SBPE approach has thus leaned towards use of established scenario planning approaches (see van der Heijden, 2005) to model plausible possible configurations of such external factors. In conventional usage of Scenario Planning, a ‘scenario’ (van der Heijden, 2005) is designed to capture aspects over which the firm has limited or no control, such as future commodity prices, costs, tax environment and exchange rates (‘externalities’). Relevant externalities may also include evolution of social, political and other factors that cannot easily be quantified. Of course, such scenarios must be regarded as plausible futures, not deterministic predictions. The importance of the scenarios lies in their use to test the robustness of strategy, not in their prediction accuracy. An example set of mining scenarios developed by the World Economic forum and can be accessed at <http://www.weforum.org/videos/mining-metals-scenarios-2030>.

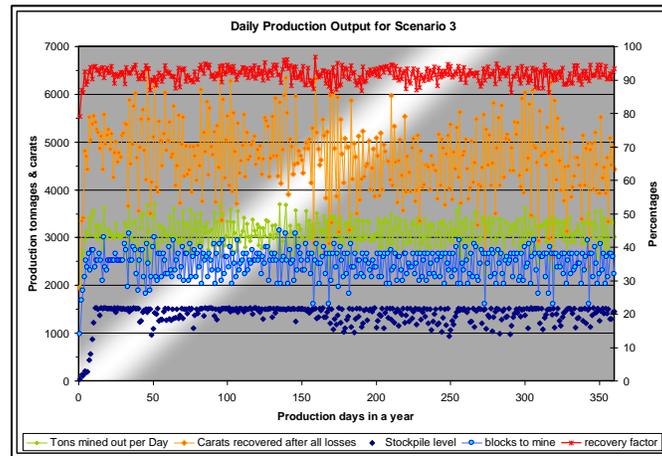


Figure 5: Key plant outputs from the integrated process simulation

Many quantitative models have been developed for interest rate and foreign exchange rates, ranging from simple extensions of Black and Scholes (1973) through Vasicek (1997) and the latest models with stochastic volatility e.g., Garman and Kohlhagen (1983). Hughston (1996) provides a good overview of the subject.

Conceptually it is possible to constrain the values for parameterising these models given the requirement for internal consistency in each of the scenarios developed. For instance in a scenario with high uncertainty of the oil price, the volatility in the forward models used to simulate oil price could be increased, and in a scenario where oil supply had been perceived to be stable the volatility parameters could be reduced. These time series realisations of input costs, prices for commodities and exchange rates can be selected at each time period when running the value chain simulation and be incorporated into the modelling. In this way the uncertainty for the external context in each of the selected scenarios can be incorporated into the financial evaluation of the project.

Simulation of the Integrated Value Chain

To derive any value from the integrated, holistic model of the value chain requires a simulation of its operation. Early work (Dowd, 1976, Dumay, 1981, Chica-Olmo, 1983, Fouquet De, 1985) focused on understanding the influence of technical aspects related to complex mining constraints and on quality control during production. As computer power increased, more simulations were run and different types of simulation methods were developed.

Simulating the integrated model is computationally intensive requiring a linking at SMU scale of the multiple geostatistical realisations with the processes carried out in mining and treatment. Increasingly faster computer hardware, combined with efficient programming (especially parallel processing) and more efficient algorithms, (e.g., direct block simulation – see Emery,(2009)), have however made generation and processing of large multi-variable Conditionally Simulated (CS) models viable. There is ongoing work that proposes (Armstrong et al., 2010) reduction of the input set of realisations in order to decrease the computational overheads of post-processing CS models, but where possible we argue that such reduction should be avoided until the *end* of the value chain, or at least as far down the value chain where the solution remains tractable. This will preserve the full uncertainty model for critical steps early in the value chain (e.g. mining depletions and stockpiling)

RESULTS AND DISCUSSION

The SBPE approach suggested here has been applied over several years to a number of operations producing a number of different products, including diamonds, gold, base metals, uranium and iron ore. Two distinct approaches have emerged:

- Operations that require the maximum recovery of the material of interest e.g. gold, diamonds. In these type of problems block selection for processing is driven by a value maximising functions; and
- Operations that require a product that is constrained by a quality target e.g., iron ore and uranium. The target is often multivariate and the selection of blocks thus requires a multivariate weighted measure of the ‘distance’ of contained grades to the target grade.

The generation of alternatives to be considered is, much like the derivation of scenarios, a process of determining the drivers that the project team would most like to explore. This is not necessarily an $N \times N$ matrix where all levels of all settings are compared against one another, but requires rather a distinctly orthogonal set of configurations that is set up to explore the outcome space. In most cases to date, the outcomes of one or two alternatives from an initial input set of 20-50 are clearly more desirable and these are then evaluated further by generating alternatives that have similar properties (i.e. map out the adjacent ‘alternative space’).

A diamond example

As an example, in a diamond project it was an option to select one of three distinct drilling densities to acquire geological (geometry) and grade data and it was possible to evaluate the differences in the valuation of the project that would arise using this methodology. The primary impact of the variable mineralisation geometry, given the tonnage constraints imposed by the operation infrastructure, reduced the throughput and recovery efficiency in the early life of the operation. By using the SBPE approach described above, the value derived from modelling the value chain based on different levels of information could be calculated. Figure 6 shows the average valuation arising from the value chain based on the three different simulated geometries.

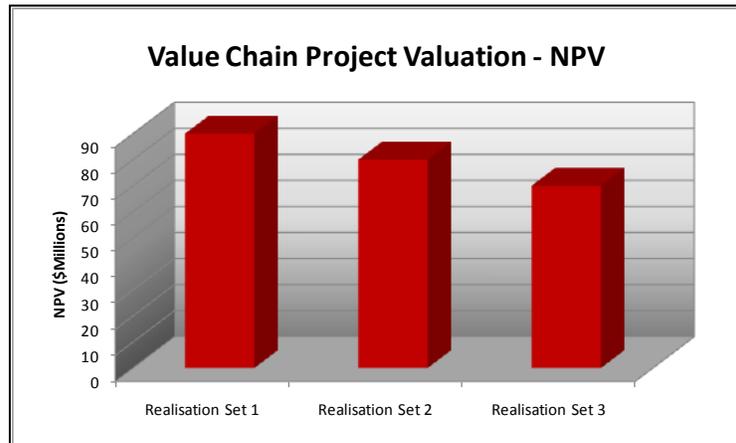


Figure 6: Plot showing average project valuation for three different simulated geometries

Value chain modelling of mineral projects using the SBPE approach requires inputs that incorporate uncertainty and variability (geostatistical simulations), and specification of systems constraints and interactions in the mining and processing steps. Once configured, the model can be run to produce outputs that correctly reflect the cumulative effect of interaction, between the variable and uncertain resources and multiple potentially biasing constraints in the system.

Computational aspects

The value chain models for mining projects are required to process a large number of blocks – in a recent case over seven million blocks containing estimates for 26 variables was provided as the input, thus computing considerations are not trivial. The value chain modelling system described has been developed in a visual basic and .net environment using MVC (Model View Controller) architecture. Data is stored and processed in SQL server running on an industrial size server. The user interface is web based that allows for multiple user configurations and the ability to generate multiple outputs. Once configured the system allows for the selection and setting of several transfer functions, capacity constraints and objective functions. Typically, these models take in the region of 8 to 12 hours to run through a single alternative configuration, allowing most of the processing to be carried out overnight.

Multiple outputs can be somewhat overwhelming to analyse and thus require the user to contemplate outputs as probabilistic. As an example, Figure 7 depicts the grade range for the output of processing 100 realisations through a value chain to produce a number of product stockpiles. The dark parallel lines show the stockpile target grade range, and the grey overlay shows the range of grades that can be expected for each stockpile's grade. The light grey represents the maximum to minimum grade, the overlaid darker grey shows the P90 to P10 distribution, and the white line shows the P50 for the output stockpile grade.

As computing power increases, it is expected that the technical limit of the approach will expand. In the interim, a trade-off has to be struck between the resolution of output and the model run time. Thus, work on scenario reduction will add further to the value to this approach.

In some of the cases considered, algorithms that model the shorter time-scale optimisation, which typically occurs in operations as new information becomes available, are required.

Implementation of these has not met with required performance in real-time. Further research is required to test ways of incorporating such algorithms.

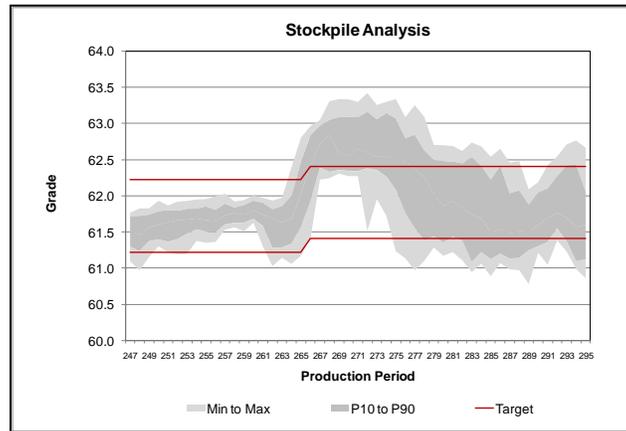


Figure 7: Summary plot of the outcome of running 100 realisations of the mineralisation through a value chain model to produce product stockpiles.

CONCLUSIONS

The mining business can be modelled as a value chain facilitating an analysis of the interaction of variability and uncertainty with system constraints. These interactions may give rise to non-linear responses that will not be correctly reflected by using traditional approaches to project evaluation. Mineral resource uncertainty can be adequately captured using a suite of geostatistical conditional simulations, with each realisation in the set reflecting the expected resource variability.

Some of the models used in the design and simulation of mining and mineral processing can be used to develop reified transfer functions. The calibrated parameterised transfer functions not only facilitate rapid simulation of complex interconnected processes but in some cases the uncertainty in the parameters of the transfer functions can be derived either from sample data, or from population balance models based on the data, and facilitate the incorporation of process efficiency uncertainty into the value chain model.

The futures in which long life mines will operate cannot be easily forecast. The paradigm of testing alternative project configurations in a few carefully crafted, internally consistent scenarios provide an alternative pathway to develop projects with a more resilient design. This is achieved by iteratively adapting design configurations that have been optimised for the singular 'corporate future' until they have an acceptable probability of success in all of the contemplated future contexts.

Scenario-based project evaluation is a value chain approach to mine design, operation and evaluation, which presents mining companies with an additional insights into their projects. In some instances significant shift in expected project value can result. However, SBPE will allow for the valuation of reduction in uncertainty even in cases where the expected value of the 'base' or 'reference' case does not shift as a result of a risk mitigation or opportunity exploiting action. This suggests that projects evaluated using the SBPE approach should produce outcomes that are more robust than those using a traditional expected outcome approach.

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